UNIT-III
DISTRIBUTIONS

BINOMIAL DISTRIBUTION

BERNOULLI TRIAL:

A trial satisfying following condition is called Bernoulli trial.
i) There are only two possible outcomes for each trial called success, failure.
ii) The probability of success is same for each trial.
iii) n trials are independent.

* Binomial distribution was discrete random variable in Bernoulli trial.
* Probability of success – p, probability of failure – q then p + q = 1
* Suppose there are n trials. Probability of getting r success, n – r failure is $p^r q^{n-r}$
* Probability of getting r success, n – r failures among n trials

$P(r) = {}^nC_r = p^r q^{n-r}$

$B(x,n,p) = {}^nC_r p^r q^{n-r}$

MEAN & VARIANCE OF BINOMIAL DISTRIBUTION:

MEAN:

$\mu = E(X) = \sum_{r=0}^{n} r\big(p(r)\big) = \sum_{r=0}^{n} r \times {}^nC_r p^r q^{n-r}$ $\boxed{{}^nC_1 = n; {}^nC_2 = \frac{n(n-1)}{2}}$

$= 0 \times q^n + 1 \times {}^nC_1 pq^{n-1} + 2\,{}^nC_2 p^2 q^{n-2} + {}^nC_3 p^3 q^{n-3} \ldots . n.p^n$

$= npq^{n-1} + 2\frac{n(n-1)}{2!}p^2 q^{n-2} + 3\frac{n(n-1)(n-2)}{3!} + \cdots np^n = np[q^{n-1} + (n-1)pq^{n-2} + \cdots + p^{n-1}] = np(q+p)^{n-1} = np$ [q + p = 1]

VARIANCE:

Variance $\sigma^2 = V(X)$

$E(X^2) - \big(E(X)\big)^2 = \sum_{r=0}^{n}[r^2 p(r)] - \mu^2$

$= \sum_{r=0}^{n}(r(r-1)+r)\,p(r) - \mu^2$ $\boxed{\sum_{r=0}^{n} rp(r) = \mu}$

$= \sum_{r=0}^{n} r(r-1)p(r) + \sum_{r=0}^{n} r\,p(r) - \mu^2 = \sum_{r=0}^{n} r(r-1)\,{}^nC_r p^r q^{n-r} + \mu - \mu^2$

$= [0 + 0 + 2\,{}^nC_2 p^2 q^{n-2} + 3.2\ {}^nC_3 p^3 q^{n-3} + \cdots + n(n-1)p^n] + \mu - \mu^2$

$= \left[2.\frac{n(n-1)}{2!}p^2 q^{n-2} + 6.\frac{n(n-1)(n-2)}{3!}p^3 q^{n-3} + \cdots + n(n-1)p^n\right] + \mu - \mu^2$

$= n(n-1)p^2[q^{n-2} + (n-2)pq^{n-3} + \cdots + p^{n-2}] + \mu - \mu^2$

$= n(n-1)p^2(q+p)^{n-2} + \mu - \mu^2 = n(n-1)p^2(1) + \mu - \mu^2$

$= n(n-1)p^2 + np - n^2p^2$

$= n^2p^2 - np^2 + np - n^2p^2 = -np^2 + np = np(1-p)$

Variance $= npq$

MODE:

Mode is the value of x for which p(x) has maximum value.
Mode is integral part of (n+1)P, if (n+1)P is not integer.
= (n + 1)P and (n+1)(p – 1) if (n +1)P is integer.

RECURRENCE RELATION FOR THE BINOMIAL DISTRIBUTION:

$p(r) = {}^nC_r p^r q^{n-r}$

$p(r+1) = {}^nC_{r+1} p^{r+1} q^{n-r-1}$

$\frac{p(r+1)}{p(r)} = \frac{{}^nC_{r+1}}{{}^nC_r}.\frac{p^{r+1}q^{n-r-1}}{p^r q^{n-r}} = \frac{n-r}{r+1}\frac{p}{q}$

$p(r+1) = \left(\frac{n-r}{r+1} \times \frac{p}{q}\right) P(r)$

BINOMIAL FREQUENCY DISTRIBUTION:

If n independent trials constitute one experiment and this experiment is repeated N times then the frequency of r success is $N \times {}^{n}C_r p^r q^{n-r}$. Since the probability of 0, 1, 2 ….r….n success in n trials are given by the terms of binomial expansion of $(q+p)^n$ therefore in N set of n trials theoretical frequencies of 0, 1, 2, …r…n success will be given by the terms of expansion of $N(p+q)^n$. The possible No. of success and their frequencies is called Binomial frequency distribution.

PROBLEMS:

1. Assume that 50% of engineering students are good in mathematics. Determine probability that is exactly 10 are good in mathematics.

Sol: Given, $p = 50\% = \frac{50}{100} = \frac{1}{2},\ q = 1 - p = \frac{1}{2}$

In Binomial, $P(r) = {}^{n}C_r p^r q^{n-r}$

$P(10) = {}^{18}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{8} = 0.1669$

2. During a stage in the manufacturing of IC's a coating must be applied. If 70% of IC received thick enough coating, find probability among 15 chips exactly 8 has thick enough coating.

Sol: Given, $p = 70\% = \frac{70}{100} = \frac{7}{10};\ \ q = \frac{3}{10}; n = 15$

In Binomial

$P(r) = {}^{n}C_r p^r q^{n-r}$

$P(x = 8) = {}^{15}C_8 \left(\frac{7}{10}\right)^{8} \left(\frac{3}{10}\right)^{7} = 0.0811$

3. Ten coins are thrown simultaneously. Find the probability of getting at least seven heads.

Sol. p = probability of getting head = ½

q = probability of getting tail = ½

Probability of getting r heads in a throw of 10 coins

$p(r) = {}^{10}C_r \left(\frac{1}{2}\right)^{r} \left(\frac{1}{2}\right)^{10-r}$

Probability of getting at least seven heads is given by

$P(X \geq 7) = P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)$

$= {}^{10}C_7 \left(\frac{1}{2}\right)^{7} \left(\frac{1}{2}\right)^{3} + {}^{10}C_8 \left(\frac{1}{2}\right)^{8} \left(\frac{1}{2}\right)^{2} + {}^{10}C_9 \left(\frac{1}{2}\right)^{9} \left(\frac{1}{2}\right)^{1} + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10}$

$= \frac{1}{2^{10}}\left[{}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right] = \frac{1}{2^{10}}[120 + 45 + 10 + 1] = \frac{176}{1024} = 0.171$

4. Ten coins are tossed simultaneously (or) one coin is tossed ten times. Find the probability of getting at least 7 heads.

Sol: Same as above

5. In 256 sets of 12 tosses of a coin in how many cases one can expect 8 heads and 4 tails.

Sol: $p = \frac{1}{2},\ q = \frac{1}{2}$

The probability of getting 8 heads and 4 tails in 12 trails is

$P(X = 8) = {}^{12}C_8 \left(\frac{1}{2}\right)^{8} \left(\frac{1}{2}\right)^{4} = \frac{12\times 11\times 10\times 9}{4\times 3\times 2\times 1}\left(\frac{1}{2^{12}}\right) = \frac{495}{2^{12}} = 0.12085$

Expected number of such cases in 256 sets

$= 256P(X = 8) = 256 \times 0.12085 = 30.93 \cong 31.$

6. Determine the probability of getting 9 exactly twice in 3 throws with a pair of dice.

Sol: Given that n = 3

p = probability of getting 9 where two dice are rolled $= \frac{4}{36}$

$B(r, m, p) = B\left(2, 3, \frac{4}{36}\right) = {}^3C_2\left(\frac{4}{36}\right)^2\left(1 - \frac{4}{36}\right) = 3\left(\frac{4}{36}\right)^2 \times \frac{8}{9} = \frac{8}{243}$

7. In eight throws of a dice 5 or 6 considered as success. Find the mean number of success and S.D.

Sol: Probability of getting 5 or 6 $= \frac{2}{6} = \frac{1}{3}$

$\Rightarrow p = \frac{1}{3}$, $q = 1 - \frac{1}{3} = \frac{2}{3}$

No. of trials $n = 8$

For Binomial distribution

Mean $= np = 8 \times \frac{1}{3} = \frac{8}{3}$

Variance $= npq = 8 \times \frac{1}{3} \times \frac{2}{3} = \frac{16}{9}$

$\Rightarrow \ \sigma^2 = \frac{16}{9} \ \Rightarrow \ \sigma = \frac{4}{3}$

8. Two dice are thrown 5 times. If getting a double (equal numbers) is success. Find the probability of success i) at least once ii) two times

Sol: $n = 5$

$S = 6^2 = 36$

Favourable events $P = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$p = \frac{6}{36} = \frac{1}{6}; \ q = \frac{5}{6}$

i) Probability of getting double at least once

$P(X > 1) = 1 - P(X = 0) = 1 - \left[{}^5C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^5\right] = 1 - \frac{5^5}{6^5} = \frac{4651}{7776} = 0.5981$

ii) $P(X = 2) = {}^5C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = 10 \times \frac{1}{36} \times \frac{125}{216} = \frac{1250}{7776} = 0.16075$

9. Determine Mode of the binomial distribution, for which mean is 4 and variance is 3.

Sol: Mean = 4 $\Rightarrow np = 4$

Variance = 3$\Rightarrow npq = 3$

$\frac{npq}{np} = \frac{3}{4} \ \Rightarrow q = \frac{3}{4}$

$\Rightarrow p = 1 - q = 1 - \frac{3}{4} = \frac{1}{4}$

Substitute the values of p and q in np = 4, we get, $n = \frac{4}{p} = \frac{4}{\frac{1}{4}} = 16$

Mode $= (n + 1)P = \frac{16+1}{4} = \frac{17}{4} = 4.25 \ \textit{not an integer}$

$[4.25] \ = \ 4$

10. Out of 800 families with 5 children each, how many would you expect to have a) 3 boys, b) 5 girls, c) either 2 or 3 boys (Assume equal probability for boys and girls)

Sol: No. of families = 800

Probability of each boy $p = \frac{1}{2}$

$q = \frac{1}{2}$

$n = 5$

$p(r) = {}^nC_r p^r q^{n-r}$

a) $P(3 \text{ boys}) = P(r = 3) = {}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = {}^5C_3\frac{1}{2^5} = \frac{10}{32} = \frac{5}{16} \ \textit{per family}$

$800\ families$, the probability of no. of familes having 3 sons $= \frac{5}{16} \times 800 = 250$

b) $P(5\ girls) = P(X = 5) = \frac{1}{32}\ per\ family$

For 800 families the probability is $\frac{1}{32} \times 800 = 25$

c) $\mathrm{P}(2 \text{ or } 3) = \mathrm{P}(\mathrm{r} = 2) + P(r = 3) = \frac{20}{32} = \frac{5}{8}\ per\ family$

For 800 families the probability is $= \frac{5}{8} \times 800 = 500$.

11. Mean and Variance of binomial distribution are 4 and 4/3. Find $\mathrm{P}(\mathrm{X} \geq 1)$.

Sol: Given $\mathrm{np} = 4$; $\mathrm{npq} = \frac{4}{3}$

$\frac{\mathrm{npq}}{\mathrm{np}} = \frac{\frac{4}{3}}{4} = \frac{1}{3}$

$\Rightarrow \mathrm{q} = \frac{1}{3}$

$\Rightarrow \mathrm{p} = 1 - \mathrm{q} = 1 - \frac{1}{3} = \frac{2}{3}$

$\Rightarrow \mathrm{n} = \frac{4}{\mathrm{p}} = \frac{4}{\frac{2}{3}} = 6$

$\mathrm{P}(\mathrm{X} \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 1 - \left(\frac{1}{3}\right)^6 = 0.9986$

12. Mean and Variance are 16, 8. Find $\mathrm{P}(\mathrm{X} \geq 1)\ and\ P(X > 2)$

Sol: Given $\mathrm{np} = 16$ and $\mathrm{npq} = 8$

$\Rightarrow \frac{\mathrm{npq}}{\mathrm{np}} = \frac{8}{16} = \frac{1}{2} \Rightarrow q = \frac{1}{2} \Rightarrow p = \frac{1}{2}\ and\ n = 32$

$\mathrm{P}(\mathrm{X} \geq 1) = 1 - \mathrm{P}(\mathrm{X} < 1) = 1 - {}^{32}\mathrm{C}_0 \left(\frac{1}{2}\right)^{32} = 1 - \frac{1}{2^{32}}$

$\mathrm{P}(\mathrm{X} > 2) = 1 - \mathrm{P}(\mathrm{X} \leq 2) = 1 - \big(\mathrm{P}(\mathrm{X} = 0) + P(X = 1) + P(X = 2)\big)$

$= 1 - \left(\frac{1}{2}\right)^{32} (1 + 32 + 496) = 0.9999$

13. The mean of binomial distribution is 3 and variance is 9/4. Find value of n, $\mathrm{P}(\mathrm{X} \geq 7)$ and $\mathrm{P}(1 \leq \mathrm{X} < 6)$

Sol: Given $\mathrm{np} = 3, \mathrm{npq} = 9/4$

$\mathrm{n} = 12$

$\mathrm{P}(\mathrm{X} \geq 7) = 1 - \mathrm{P}(\mathrm{X} < 7) = 1 - \big(\mathrm{P}(\mathrm{X} = 0) + \cdots \mathrm{P}(\mathrm{X} = 6)\big)$

$= 1 - 0.8554 = 0.1446$

$\mathrm{P}(1 \leq \mathrm{X} < 6) = \mathrm{P}(\mathrm{X} = 1) + \cdots .\mathrm{P}(\mathrm{X} = 5) = \frac{37}{412} = 0.82$

14. The mean and variance of a binomial distribution are 6 and 3 respectively. Find mode of binomial distribution.

Sol: Mean $\mathrm{np} = 6$

Variance $= \mathrm{npq} = 3$

$\frac{\mathrm{npq}}{\mathrm{np}} = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow q = \frac{1}{2} \Rightarrow p = \frac{1}{2}$

$\mathrm{n} = \frac{6}{\mathrm{p}} = \frac{6}{\frac{1}{2}} = 12$

Mode of binomial distribution $(\mathrm{n} + 1)\mathrm{P} = \frac{(12+1)1}{2} = \frac{13}{2} = 6.5$

Since (n + 1) P is not an integer, integral part of 6.5 is mode

$\therefore$ Mode = 6

15. Probability of a man hitting a target is 1/3.
If he fires 5 times what is the probability of his hitting the target at least twice.

Sol: $p = \frac{1}{3},\ n = 5, q = \frac{2}{3}$

Probability of hitting the target at least twice

$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X = 0) + P(X = 1)$

$= 1 - \left[{}^5C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^5 + {}^5C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^4\right] = 1 - \left[\frac{5\times2^5}{3^5} + \frac{5\times2^4}{3^5}\right] = \frac{1}{81}$

16. In a binomial distribution consisting of 5 independent trials. Probability of 1 and 2 success are 0.4096 and 0.2048. Find the parameter p of the distribution.

Sol: $n = 5$

$P(X = 1) = 0.4096,\ P(X = 2) = 0.2048$

${}^5C_1 p^1 (1 - p)^4 = 0.4096;\ \ {}^5C_2 p^2 (1 - p)^3 = 0.2048$

$\frac{{}^5C_1 p^1 (1-p)^4}{{}^5C_2 p^2 (1-p)^3} = \frac{0.4096}{0.2048} \Rightarrow \frac{5(1-p)}{10p} = 2$

$\Rightarrow 1 - p = 4p \Rightarrow 5p = 1 \Rightarrow p = \frac{1}{5} = 0.2$

$\Rightarrow q = 1 - p = 1 - 0.2 = 0.8$

17. 20% of items produced from a factory are defective. Find the probability that in a sample of 5 chosen at random

i) None is defective ii) One is defective iii) $P(1 < x < 4)$

Sol: As per probability of defective item

$p = \frac{20}{100} = \frac{1}{5};\ q = \frac{4}{5}; n = 5$

i) $P(X = 0) = {}^5C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^5 = \left(\frac{4}{5}\right)^5$

ii) $P(X = 1) = \left(\frac{4}{5}\right)^4$

iii) $P(1 < X < 4) = P(X = 2) + P(X = 3) = 10\left(\frac{4^2}{5^4}\right)$

18. Find maximum n such that the probability of getting no heads in tossing a coin n times is greater than 0.1

Sol: $p = \frac{1}{2};\ n = n$

$P(X = 0) > 0.1$

${}^nC_0 (p)^0 (1 - p)^n > \frac{1}{10}$

$\left(\frac{1}{2}\right)^n > \frac{1}{10}$

$n = 1, \frac{1}{2} = 0.5,\ \frac{1}{2^2} = 0.25$

$n = 3, \frac{1}{2^3} = 0.125,\ \frac{1}{2^4} = 0.0625$

for $n \geq 4$ probability is $< \frac{1}{10}$

$\therefore n = 3$

19. Fit a binomial distribution to the following data.

n	0	1	2	3	4	5
f	2	14	20	34	22	8

n is no. of trials.

Sol: In fitting binomial distribution first of all mean, variance of the data are equated to np and npq. The expected frequencies are calculated from these values of n and p.

Here n = 5

N = Total frequency $\sum_{i=0}^{5} f_i = 2 + 14 + 20 + 34 + 22 + 8 = 100$

$\mu = np = \frac{\sum f_i x_i}{N} = \frac{0(2)+1(14)+2(20)+3(34)+4(22)+5(8)}{100} = \frac{284}{100} = 2.84$

$np = 2.84$

$\Rightarrow p = \frac{2.84}{5} = 0.568$

$\Rightarrow q = 1 - p = 1 - 0.568 = 0.432$

Binomial distribution is given by P(r) = $^nC_r(p)^r(q)^{n-r}$

r	$\mathbf{P(r) = {}^nC_r(p)^r(q)^{n-r}}$	**Expected frequency** $\boldsymbol{N \times P(r)}$
0	$^5C_0(0.568)^0(0.432)^5 = 0.150$	$100 \times 0.150 = 1.5 \cong 1$
1	$^5C_1(0.568)^1(0.432)^4 = 0.0989$	$100 \times 0.989 = 9.89 \cong 10$
2	$^5C_2(0.568)^2(0.432)^3 = 0.260$	$100 \times 0.260 = 26$
3	$^5C_3(0.568)^3(0.432)^2 = 0.341$	$100 \times 0.341 = 34$
4	$^5C_4(0.568)^4(0.432)^1 = 0.224$	$100 \times 0.224 = 22$
5	$^5C_5(0.568)^5(0.432)^0 = 0.059$	$100 \times 0.59 = 5.9$

x_i	0	1	2	3	4	5
f_i	2	14	20	34	22	8
Expected frequency	1	10	26	34	22	6

20. Fit a binomial distribution to the following frequency data.

n	0	1	2	3	4
f	28	62	46	10	4

n is no. of trials.

Sol: Here n = 5

$\sum f_i = 150$

$Mean\ \mu = np = \frac{\sum f_i x_i}{\sum f_i} = \frac{0(28)+1(62)+2(46)+3(10)+4(4)}{150} = \frac{200}{150} = 1.33$

$np = 1.33$

$\Rightarrow p = \frac{1.33}{5} = 0.266$

$\Rightarrow q = 1 - p = 1 - 0.266 = 0.734$

Binomial distribution is given by

$B(r, n, p) = {}^nC_r(p)^r(q)^{n-r}$

$B(0, 5, 0.266) = {}^5C_0(0.266)^0(0.734)^5$

$p(0) = 0.213$

$p(1) = 0.3857$

$p(2) = 0.2765$

$p(3) = 0.0968$

$p(4) = 0.0183$

Expected frequency $N \times p(x)$

$f(0) = N \times p(0) = 150(0.213) = 31.9 \cong 32$

$f(1) = 57.8 \cong 58$

$f(2) = 41.47 \cong 41$

$f(3) = 14.52 \cong 15$

$f(4) = 2.74 \cong 3$

x_i	0	1	2	3	4
f_i	28	62	46	10	4
Expected frequency	32	58	41	15	3

POISSON DISTRIBUTION:

Poisson distribution is special case in binomial distribution. It can be derived from binomial distribution when p is very small, n is very large, $np = \lambda$ is finite.

* In the binomial distribution probability of r success

$$p(r) = {}^{n}C_r (p)^r (q)^{n-r} = \frac{n(n-1)(n-2)....(n-(r-1))}{r!} p^r (1-p)^{n-r}$$

$$= \frac{n(n-1)(n-2)....(n-(r-1))}{r!} p^r \frac{(1-p)^n}{(1-p)^r}$$

Put $np = \lambda \; then \; n = \frac{\lambda}{p}$

$$p(r) = \frac{\frac{\lambda}{p}\left(\frac{\lambda}{p}-1\right)\left(\frac{\lambda}{p}-2\right)....\left(\frac{\lambda}{p}-(r-1)\right)}{r!} p^r \frac{(1-p)^n}{(1-p)^r} = \frac{\lambda(\lambda-p)(\lambda-2p)....(\lambda-p(r-1))}{r!p^r} p^r \frac{(1-p)^n}{(1-p)^r}$$

$as \; n \to \infty, p \to 0$,we have

$$p(r) = \frac{\lambda.\lambda.......r \; times}{r!} \lim_{n\to\infty} \left(1 - \frac{\lambda}{n}\right)^n \lim_{p\to 0} \frac{1}{(1-p)^r} = \frac{\lambda^r}{r!} \lim_{n\to\infty} \left[\left(1 - \frac{\lambda}{n}\right)^{\frac{n}{-\lambda}}\right]^{-\lambda}$$

$$= \frac{\lambda^r}{r!} e^{-\lambda} \left[since \; \lim_{n\to\infty} \left[\left(1 - \frac{\lambda}{n}\right)^{\frac{n}{-\lambda}}\right] = e\right]$$

Probability of r success in Poisson distribution $p(r) = \frac{\lambda^r}{r!} e^{-\lambda}$

* A random variable X is said to follow Poisson distribution if it assumes only non – negative values and its probability distribution is given by

$$P(x, \lambda) = P(X = x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & x = 0, 1, 2, ... \\ 0 & otherwise \end{cases}$$

Sum of Poisson probabilities is 1

i.e.,$p(0) + p(1) + \cdots .. = e^{-\lambda} + \frac{e^{-\lambda}\lambda}{1!} + \cdots$

$$= e^{-\lambda}\left(1 + \frac{\lambda}{1} + \frac{\lambda^2}{2!} + \cdots\right) = e^{-\lambda}e^{\lambda} = 1$$

MEAN& VARIANCE OF POISSON DISTRIBUTION:

Mean$\mu = E(x) = \sum_{r=0}^{\infty} r\big(p(r)\big) = \sum_{r=0}^{\infty} r.\frac{e^{-\lambda}\lambda^r}{r!}$

$$= \sum_{r=0}^{\infty} r.\frac{e^{-\lambda}\lambda^r}{r(r-1)!} = e^{-\lambda} \sum_{r=0}^{\infty} \frac{\lambda^r}{(r-1)!}$$

$$= e^{-\lambda}\left(\frac{\lambda}{0!} + \frac{\lambda^2}{1!} + \frac{\lambda^3}{2!} + \cdots ...\right) = \lambda e^{-\lambda}\left(1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots ...\right) = \lambda e^{-\lambda}e^{\lambda}$$

$\Rightarrow Mean \; \mu = \lambda$

Variance: $\sigma^2 = E(X^2) - \big(E(X)\big)^2 = \sum_{r=0}^{\infty} r^2 p(r) - \mu^2 = \sum_{r=0}^{\infty} \frac{r^2 e^{-\lambda}\lambda^r}{r!} - \mu^2$

$$= e^{-\lambda}\left[\sum_{r=0}^{\infty} r^2 \frac{\lambda^r}{r!}\right] - \mu^2 = e^{-\lambda}\left[\frac{1.\lambda}{1!} + \frac{\lambda^2 2^2}{2!} + \frac{\lambda^3 3^3}{3!} + \cdots\right] - \lambda^2$$

$$= e^{-\lambda}.\lambda\left[1 + 2\lambda + \frac{3\lambda^2}{2!} + \frac{4\lambda^3}{3!} + \cdots ..\right] - \lambda^2$$

$$= e^{-\lambda}.\lambda\left[1 + \frac{(1+1)\lambda}{1!} + \frac{(1+2)\lambda^2}{2!} + \frac{(1+3)\lambda^3}{3!} + \cdots\right] - \lambda^2$$

$$= e^{-\lambda}.\lambda\left[1+\frac{\lambda}{1!}+\frac{\lambda^2}{2!}+\frac{\lambda^3}{3!}+\cdots..\right]+\left[\frac{\lambda}{1}+\frac{2\lambda^2}{2!}+\frac{3\lambda^3}{3!}+\cdots.\right]-\lambda^2$$
$$= e^{-\lambda}.\lambda\left[e^{\lambda}+\lambda\left[1+\frac{\lambda}{1!}+\frac{\lambda^2}{2!}+\cdots.\right]\right]-\lambda^2 = e^{-\lambda}.\lambda\left[e^{\lambda}+\lambda e^{\lambda}\right]-\lambda^2$$
$$= \lambda e^{-\lambda}.e^{\lambda}+\lambda^2 e^{-\lambda}e^{\lambda}-\lambda^2 = \lambda+\lambda^2-\lambda^2 \Rightarrow \sigma^2=\lambda$$
$$\Rightarrow Variance\ \sigma^2=\lambda$$

MODE OF THE POISSON DISTRIBUTION:

Mode value of r for which p(r) is maximum.
Mode of Poisson distribution lies between $(\lambda-1)\ and\ \lambda$

Case i) If λ is an integer then $\lambda-1$ is also integer. So we have two maximum values and the distribution is bi – modal.

Case ii) If λ is not an integer, the mode of Poisson distribution is integral part of λ.

RECURRENCE RELATION FOR POISSON DISTRIBUTION:

$$p(r)=\frac{e^{-\lambda}\lambda^r}{r!}$$
$$p(r+1)=\frac{e^{-\lambda}\lambda^{r+1}}{(r+1)!}=\frac{\lambda}{r+1}.\frac{e^{-\lambda}\lambda^r}{r!}$$
$$p(r+1)=\frac{\lambda}{r+1}p(r)$$

PROBLEMS:

1. In Poisson if $P(x=2)=P(x=3)$ then find variance of x and P(4)

Sol: Given, P(x = 2) = P(x = 3)
In Poisson distribution,
$$P(x)=\frac{e^{-\lambda}\lambda^x}{x!}$$
i.e.,$\frac{e^{-\lambda}\lambda^2}{2}=\frac{e^{-\lambda}\lambda^3}{6}\Rightarrow\ \lambda=3$
Variance $=\lambda=3$
$$P(4)=\frac{e^{-\lambda}\lambda^4}{24}=\frac{e^{-3}(3)^4}{24}=0.168$$

2. 0.8% of fuses delivered to a company are defective. Use Poisson approximation to determine the probability that 4 fuses will be defective in random sample of 400.

Sol: Given, $p=0.8\%=\frac{0.8}{100}=0.008,\ n=400$
Mean $\lambda=np=400\times0.008=4\times0.8=3.2$
In Poisson,
$$P(x)=\frac{e^{-\lambda}\lambda^x}{x!}=P(x=4)=\frac{e^{-(3.2)}(3.2)^4}{24}=0.1781$$

3. At a check counter customers arrive on average 1.5 per minute. Find probability that in any given minute (i) At most 4, (ii) Exactly 4, (iii) At least 4 will arrive.

Sol: Given, Mean or Average, $\lambda=1.5$
In Poisson, $P(x)=\frac{e^{-\lambda}\lambda^x}{x!}$
i) At most 4:
$$P(x\le4)=P(0)+P(1)+P(2)+P(3)+P(4)$$
$$=e^{-\lambda}\left[\frac{\lambda^0}{1}+\frac{\lambda^1}{1!}+\frac{\lambda^2}{2!}+\frac{\lambda^3}{3!}+\frac{\lambda^4}{4!}\right]=e^{-1.5}\left[1+1.5+\frac{(1.5)^2}{2}+\frac{(1.5)^3}{6}+\frac{(1.5)^4}{24}\right]=0.98$$
ii) Exactly 4:

$P(x=4)=\frac{e^{-1.5}(1.5)^4}{4!}=0.47$

iii) At least 4:
$P(x\geq 4)=1-P(x<4)$
$=1-P(x\leq 4)+P(4)$
$=1-0.9814+0.471=0.065$

4. In Poisson $P(x=1).\frac{3}{2}=P(x=3). Find\ P(x\geq 1)$

Sol: $\frac{e^{-\lambda}\lambda}{1!}.\frac{3}{2}=\frac{e^{-\lambda}\lambda^3}{3!}\Rightarrow 3=\frac{\lambda^2}{3}\Rightarrow \lambda^2=9\Rightarrow \lambda=3$
$P(x\geq 1)=1-P(x<1)$
$=1-P(0)=1-\frac{e^{-3}3^0}{0!}=1-e^{-3}=0.9502.$

5. 2% of items produced from a factory are defective. The items are parked in boxes what is the probability that there will be 2 defective, atleast 3 defective in a box of 100 items.

Sol: Given $p=2\%=\frac{2}{100}=0.02\ (very\ small)$
$n=100\ (large)$
Mean, $\lambda=np\ (In\ Binomial)$
$=100\times 0.02=2$
In Poisson, $P(x)=\frac{e^{-\lambda}\lambda^x}{x!}$
$P(2)=\frac{e^{-2}2^2}{2!}=0.2707$
$P(x\geq 3)=1-P(x<3)$
$=1-[P(0)+P(1)+P(2)]=1-e^{-2}\left[\frac{2^0}{0!}+\frac{2^1}{1!}+\frac{2^2}{2!}\right]=1-e^{-2}\left[3+\frac{(2)^2}{2}\right]$
$=1-5e^{-2}=0.3233.$

6. Variance of Poisson variate is 3. Find probability that (i) $x=0$, (ii) $1\leq x<4$, (iii) $0<x\leq 3$

Sol: Given, Variance, $\lambda=3, P(x)=\frac{e^{-\lambda}\lambda^x}{x!}$
(i) $x=0$
$P(x=0)=\frac{e^{-3}\lambda^0}{0!}=e^{-3}=0.0498$

(ii) $1\leq x<4$
$P(1\leq x<4)=P(1)+P(2)+P(3)=e^{-3}\left[\frac{\lambda^1}{1!}+\frac{\lambda^2}{2!}+\frac{\lambda^3}{3!}\right]=e^{-3}\left[3+\frac{9}{2}+\frac{27}{6}\right]$
$=e^{-3}[12]=0.5974.$

(iii) $0<x\leq 3$
$P(0<x\leq 3)=P(1)+P(2)+P(3)=12e^{-3}=0.5974.$

7. Fit a Poisson distribution to the following data:

x	0	1	2	3	4
f	109	65	22	3	1

Sol: Fitting Poisson distribution means finding expected frequencies.
Expected frequency $=N\times P(x)$
N = Sum of frequencies $=\sum f=200$
$P(x)=\frac{e^{-\lambda}\lambda^x}{x!}$

Mean$\lambda = \frac{\sum fx}{\sum f} = \frac{0+65+44+9+4}{200} = 0.61.$

$\boldsymbol{x}$	$\boldsymbol{P(x) = \frac{e^{-\lambda}\lambda^x}{x!}}$	**Expected frequency $\boldsymbol{N.P(x)}$**
0	$P(0) = e^{-0.61}(0.61)^0 = 0.5434$	$200 \times 0.5434 = 108.68$
1	$P(1) = e^{-0.61}(0.61)^1 = 0.3314$	$200 \times 0.3314 = 66.28$
2	$P(2) = \frac{e^{-0.61}(0.61)^2}{2} = 0.1011$	$200 \times 0.1011 = 20.22$
3	$P(3) = \frac{e^{-0.61}(0.61)^3}{6} = 0.0206$	$200 \times 0.0206 = 4.12$
4	$P(4) = \frac{e^{-0.61}(0.61)^4}{24} = 0.0031$	$200 \times 0.0031 = 0.62$

Fitted Poisson distribution:

$\boldsymbol{x}$	0	1	2	3	4
$\boldsymbol{f}$	109	65	22	3	1
$\boldsymbol{E(f)}$	109	66	20	4	1

8. If the variance of Poisson variate is 3. Find probability that i) X = 0, ii) $1 \leq X < 4$ iii) $0 < X \leq 3$

Sol: $\lambda = 3$

$p(x,\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$

i) $P(X = 0) = \frac{e^{-3}\lambda^0}{0!} = e^{-3} = 0.0498$

ii) $P(1 \leq X < 4) = P(X = 1) + P(X = 2) + P(X = 3)$

$= \frac{e^{-3}3^1}{1} + \frac{e^{-3}3^2}{2!} + \frac{e^{-3}3^3}{3!} = 3.e^{-3} + e^{-3}\frac{3^2}{2} + e^{-3}\frac{3^3}{6} = e^{-3}\left[3 + \frac{9}{2} + \frac{27}{6}\right]$

$= e^{-3}(12) = 0.5974$

iii) $P(0 < X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3) = \frac{e^{-3}3^1}{1} + \frac{e^{-3}3^2}{2!} + \frac{e^{-3}3^3}{3!}$

$= 3.e^{-3} + e^{-3}\frac{3^2}{2} + e^{-3}\frac{3^3}{6} = e^{-3}\left[3 + \frac{9}{2} + \frac{27}{6}\right] = e^{-3}(12) = 0.597$

9. The average number of phone calls / minute coming into a switch board between 2 pm and 4 pm is 2.5. Determine the probability that during one particular minute there will be i) 4 or fewer and ii) more than 6 calls.

Sol: Given $\lambda = 2.5$

$p(x) = e^{-\lambda}\frac{\lambda^x}{x!}$

i) $P(X \leq 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$= e^{-2.5}\left[\frac{(2.5)^0}{0!} + \frac{(2.5)^1}{1!} + \frac{(2.5)^2}{2!} + \frac{(2.5)^3}{3!} + \frac{(2.5)^4}{4!}\right]$

$= e^{-2.5}[1 + 2.5 + 3.125 + 2.6042 + 1.6276] = 0.8912$

10. 2% of the items of a factory are defective. The items are packed in boxes. What is the probability that there will be i) 2 defective, ii) at least three are defective in a box of 100 items.

Sol: Given p = probability of defective items

$p = 0.02,\ n = 100$

$Mean\ \lambda = np = 0.02 \times 100 = 2$

i) $P(X = 2) = \frac{e^{-2}2^2}{2!} = \frac{2}{e^2} = 0.2706$

ii) $P(X \geq 3) = 1 - [P(X=0) + P(X=1) + P(X=2)] = 1 - e^{-2}\left[\frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!}\right]$
$= 1 - e^{-2}(1+2+2) = 1 - e^{-2} = 0.3233.$

11. The average number of accidents on any day on a national highway is 1.8. Determine the probability that the number of accidents are i) at least one, ii) at most one

Sol: $\lambda = 1.8$
$P(X \geq 1) = 1 - P(X=0) = 0.8347$
$P(X \leq 1) = P(X=0) + P(X=1) = 0.4628$

12. If X is a Poisson variate $P(X=0) = P(X=1)$. $Find\ P(X=0)$ and using recurrence formula find the probabilities at x = 1, 2, 3, 4 and 5.

Sol: Given that $P(X=0) = P(X=1)$
$\frac{e^{-\lambda}\lambda^0}{0!} = \frac{e^{-\lambda}\lambda^1}{1!} \Rightarrow \lambda = 1$
i) $P(X=0) = ?$
$P(X=0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-1} = 0.3678$
ii) Recurrence formula for Poisson distribution is
$p(r+1) = \frac{\lambda}{r+1}p(r)$
$r = 0, p(1) = p(0+1) = \frac{1}{0+1}p(0) = \frac{1}{1}0.3678 = 0.3678$
$r = 1, p(2) = p(1+1) = \frac{1}{1+1}p(1) = \frac{1}{2}0.3678 = 0.1839$
$r = 2, p(3) = p(2+1) = \frac{1}{2+1}p(2) = \frac{1}{3}0.1839 = 0.0613$
$r = 3, p(4) = p(3+1) = \frac{1}{3+1}p(3) = \frac{1}{4}0.0613 = 0.01532$
$r = 4, p(5) = p(4+1) = \frac{1}{4+1}p(4) = \frac{1}{5}0.01532 = 0.00306$

13. If the variance of Poisson variate is 3, find probability that i) $X = 0$, ii) $1 \leq X < 4$, iii) $0 < X \leq 3$.

Sol: For a Poisson distribution Mean = Variance
$\lambda = 3$; $P(r, \lambda) = \frac{e^{-\lambda}\lambda^r}{r!}$
$P(X=0) = \frac{e^{-3}3^0}{0!} = e^{-3} = 0.0498$
$P(1 \leq X < 4) = P(X=1) + P(X=2) + P(X=3) = e^{-3}(12) = 0.5974.$
$P(0 < X \leq 3) = P(X=1) + P(X=2) + P(X=3) = 0.5974.$

14. If X is a Poisson variate such that$3P(X=4) = \frac{1}{2}P(X=2) + P(X=0)$. Find i) Mean of X, ii) $P(X \leq 2)$

Sol: Given $3P(X=4) = \frac{1}{2}P(X=2) + P(X=0)$
$\Rightarrow 3\left[\frac{e^{-\lambda}\lambda^4}{4!}\right] = \frac{1}{2}\left[\frac{e^{-\lambda}\lambda^2}{2!}\right] + \frac{e^{-\lambda}\lambda^0}{0!}$
$\Rightarrow 3\left[\frac{e^{-\lambda}\lambda^4}{24}\right] = \frac{1}{2}\left[\frac{e^{-\lambda}\lambda^2}{2}\right] + e^{-\lambda} \Rightarrow \frac{e^{-\lambda}\lambda^4}{2} = e^{-\lambda}[\lambda^2 + 4] \Rightarrow \lambda^4$
$= 2\lambda^2 + 8 \Rightarrow \lambda^4 - 2\lambda^2 - 8 = 0$
$\Rightarrow \lambda^2(\lambda^2 - 4) + 2(\lambda^2 - 4) = 0 \Rightarrow (\lambda^2 + 2)(\lambda^2 - 4) = 0 \Rightarrow \lambda^2$
$= -2\ and\ \lambda^2 = 4 \Rightarrow \lambda = \pm 2$
ii) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$
$= \frac{e^{-2}2^0}{0!} + \frac{e^{-2}2^1}{1!} + \frac{e^{-2}2^2}{2!} = e^{-2}[1+2+2] = 4(e^{-2}) = 0.5413$

15. Fit a Poisson distribution to calculate the theoretical frequencies for the following data.

x	0	1	2	3	4
f	109	65	22	3	1

Mean $\sum \frac{fx}{N} = \frac{0\times109+1\times65+2\times22+3\times3+4\times1}{200} = \frac{122}{200} = 0.61$

$\Rightarrow \ \lambda = 0.61$

Required probability distribution $= N \times \frac{e^{-\lambda}\lambda^r}{r!} = N \times P(r)$

$= 200 \times \frac{e^{-0.61}(0.61)^r}{r!} = \frac{200\times0.5435\times(0.61)^r}{r!} = \frac{108.7\times(0.61)^r}{r!}$

r	$\boldsymbol{N \times P(r) = \frac{108.7(0.61)^r}{r!}}$	**Expected Frequency**
0	108.7	109
1	$108.7\times\ 0.61 = 66.3$	66
2	$108.7 \times \frac{(0.61)^2}{2!} = 20.2$	21
3	$108.7 \times \frac{(0.61)^3}{3!} = 4.1$	4
4	$108.7 \times \frac{(0.61)^4}{4!} = 0.7$	1

PRACTICE PROBLEM:

16. Fit Poisson distribution:

x	0	1	2	3	4	5
f(x)	42	33	14	6	4	1

NORMAL DISTRIBUTION:

Normal distribution is defined on continuous random variable.

A random variable X is said to have normal distribution if its density function or probability distribution is given by

$f(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \ -\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0$

Where $\mu = mean, \sigma = S.D.$

MEAN OF NORMAL DISTRIBUTION:

Mean $= E[X] = \int_{-\infty}^{\infty} x\, f(x)dx = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} xe^{-\frac{1}{2}\left(\frac{x-b}{\sigma}\right)^2} dx = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}(\sigma^2+b)e^{-\frac{z^2}{2}}\sigma dz$

$= \frac{\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\infty} Ze^{-\frac{z^2}{2}}dz + \frac{b}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{z^2}{2}}dz = 0 + \frac{b}{\sqrt{2\pi}} 2\int_0^{\infty} e^{-\frac{z^2}{2}}dz$

$$\left\{\begin{array}{c}
\left[Z = \frac{x-b}{\sigma}\right]; \ x = (\sigma^2+b); \ dZ = \frac{dx}{\sigma} \Rightarrow dx = \sigma dZ \ (limits\ same) \\
even\ function\ \ if\ f(-x) = f(x)\ is\ even \\
If\ f(-x) = -f(x) then\ odd\ function\ \Rightarrow for\ odd\ \int_{-\infty}^{\infty} f(x)dx = 0 \\
= Ze^{-\frac{z^2}{2}}\ is\ odd;\ \ e^{-\frac{z^2}{2}} = even \\
Let\ \frac{Z^2}{2} = t \Rightarrow Z = 2\sqrt{t} \\
\frac{2Z}{2}dZ = dt \Rightarrow dZ = \frac{dt}{Z} = \frac{dt}{2\sqrt{t}}
\end{array}\right\}$$

$where\ b = \mu\ \int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

$$= \frac{2b}{\sqrt{2\pi}}\int_0^\infty e^{-t}\frac{dt}{2\sqrt{t}} = \frac{b}{\sqrt{2\pi}}\int_0^\infty e^{-t}t^{-\frac{1}{2}}dt = \frac{b}{\sqrt{2\pi}}2\int_0^\infty e^{-t}\frac{dt}{\sqrt{2t}} = \frac{b}{\sqrt{\pi}}\int_0^\infty e^{-t}t^{-\frac{1}{2}}dt$$

$$\left\{\begin{array}{c}
\text{Gamma function } \Gamma(n) = \int_0^\infty e^{-t}t^{n-1}dn \\
\Gamma\left(\frac{1}{2}\right) = \int_0^\infty e^{-t}t^{-\frac{1}{2}}dt \\
\int_0^\infty e^{-\frac{x^2}{2}}dx = \frac{\pi}{2} \\
\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi} \\
\Gamma\left(\frac{1}{2}\right)\Gamma\left(1-\frac{1}{2}\right) = \frac{\pi}{\sin\frac{\pi}{2}} \\
\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right) = \frac{\pi}{2} \Rightarrow \left(\Gamma\left(\frac{1}{2}\right)\right)^2 = \frac{\pi}{2} \Rightarrow \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}
\end{array}\right\}$$

$$= \frac{b}{\sqrt{\pi}}\Gamma\left(\frac{1}{2}\right) = \frac{b}{\sqrt{\pi}}\sqrt{\pi} = b$$

VARIANCE OF NORMAL DISTRIBUTION:

Variance $E(x-\mu)^2 = \int_{-\infty}^{\infty}(x-b)^2 f(x)dx = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}(x-b)^2 e^{-\frac{1}{2}\left(\frac{x-b}{\sigma}\right)^2}$ $(\mu = b)$

$$= \frac{1}{\sigma\sqrt{2\pi}}\int(2\sigma + b - b)^2 e^{-\frac{1}{2}z^2}dz(\sigma) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}z^2 e^{-\frac{1}{2}z^2}dz\ (\sigma^2)\ (even\ function)$$

$$= \frac{\sigma^2}{\sqrt{2\pi}}2\int_0^\infty z^2 e^{-\frac{1}{2}z^2}dz = \frac{2\sigma^2}{\sqrt{2\pi}}\int_0^\infty 2t\, e^{-t}\frac{dt}{\sqrt{2t}} = \frac{2\sigma^2}{2\sqrt{\pi}}2\int_0 \sqrt{t}e^{-t}dt = \frac{2\sigma^2}{\sqrt{\pi}}\int_0^\infty \sqrt{t}e^{-t}dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}}\frac{1}{2}\sqrt{\pi} = \sigma^2$$

$$\left\{\begin{array}{c}
\frac{x-b}{\sigma} = z \Rightarrow x = \sigma z + b \Rightarrow dx = \sigma dz \\
let\,\frac{z^2}{2} = t \Rightarrow z^2 = 2t \Rightarrow zdz = dt \Rightarrow dz = \frac{dt}{\sqrt{2t}} \\
T(n) = \int_0^\infty e^{-t}t^n dt \\
T\left(\frac{3}{2}\right) = \int_0^\infty e^{-t}t^{\frac{3}{2}}dt = \int_0^\infty e^{-t}t^{\frac{1}{2}}dt \\
T(n) = (n-1)T(n-1) \Rightarrow T\left(\frac{3}{2}\right) = \left(\frac{3}{2}-1\right)T\left(\frac{3}{2}-1\right) = \frac{1}{2}T\left(\frac{1}{2}\right) = \frac{1}{2}\sqrt{\pi}
\end{array}\right\}$$

MEAN DEVIATION FROM THE MEAN FOR NORMAL DISTRIBUTION:

Mean distribution $= E(x-\mu) = \int_{-\infty}^{\infty}(x-b)f(x)dx = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}(x-b)^2 e^{-\frac{1}{2}\left(\frac{x-b}{\sigma}\right)^2}dx$

$$= \frac{\sigma}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}|z|e^{-\frac{z^2}{2}}\sigma dz = \frac{\sigma}{\sqrt{2\pi}}2\int_0^\infty z e^{-\frac{z^2}{2}}dz = \frac{\sqrt{2}\sigma}{\sqrt{\pi}}\int_0^\infty e^{-t}dt = \sqrt{\frac{2}{\pi}}\sigma[-e^{-t}]_0^\infty = \sqrt{\frac{2}{\pi}}\sigma$$

$$= \sqrt{\frac{14}{22}}\sigma \cong \frac{4}{5}\sigma$$

FOR MEDIAN:

Suppose M is the median of normal distribution, then

$\int_{-\infty}^{M} f(x)dx = \int_M^\infty f(x)dx = \frac{1}{2}$, $\int_{-\infty}^{M} f(x)dx = \frac{1}{2}$

$\int_{-\infty}^{b} f(x)dx + \int_b^M f(x)dx = \frac{1}{2}$ ……..(1)

Consider $\int_{-\infty}^{b} f(x)dx = \int_{-\infty}^{b}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{x-b}{\sigma}\right)^2}dx = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{0} e^{-\frac{z^2}{2}}\sigma dz = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0} e^{-\frac{z^2}{2}}dz$

$$= \frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-\frac{z^2}{2}}dx = \frac{1}{\sqrt{2\pi}}.\frac{\sqrt{\pi}}{\sqrt{2}} = \frac{1}{2}\left[since\ \int_0^\infty e^{-\frac{z^2}{2}}dz = \frac{\sqrt{\pi}}{\sqrt{2}}\right]$$

From (1) $\int_{\infty}^{b} f(x)dx + \int_{b}^{M} f(x)dx = \frac{1}{2}$

$\frac{1}{2} + \int_{b}^{M} f(x)dx = \frac{1}{2} \Rightarrow \int_{b}^{M} f(x)dx = 0 \Rightarrow M = b$

∴ Median = b.

MODE OF THE DISTRIBUTION:

Mode is value of x for which f(x) is maximum. Mode is solution of $f'(x) = 0 \text{ and } f''(x) < 0$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-b}{\sigma}\right)^2} \Rightarrow f'(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-b}{\sigma}\right)^2}\left(-\left(\frac{x-b}{\sigma}\right)\right)\frac{1}{\sigma} = -\left(\frac{x-b}{\sigma^2}\right)f(x)$$

$$f'(x) = 0$$

$$-\left(\frac{x-b}{\sigma^2}\right)f(x) = 0 \Rightarrow \left(\frac{x-b}{\sigma^2}\right) = 0 \Rightarrow x - b = 0 \Rightarrow x = b$$

$$f''(x) = -\left[\left(\frac{x-b}{\sigma^2}\right)f'(x) + f(x)\left(\frac{1}{\sigma}\right)\right]$$

$$= -\frac{1}{\sigma}\left[\left(\frac{x-\mu}{\sigma}\right)\left(-\left(\frac{x-b}{\sigma}\right)\right)f(x) + f(x)\right]$$

$$= -\frac{1}{\sigma}\left[-\frac{(x-b)^2}{\sigma^2}f(x) + f(x)\right] = -\frac{f(x)}{\sigma}\left[-\frac{(x-b)^2}{\sigma^2} + 1\right] \quad at\ x = b$$

$$at\ x = b\ ,\ f''(x) < 0\ \Rightarrow mode = b$$

∴In normal distribution Mean = Median = Mode

CHIEF CHARACTERISTIC OF THE NORMAL DISTRIBUTION:

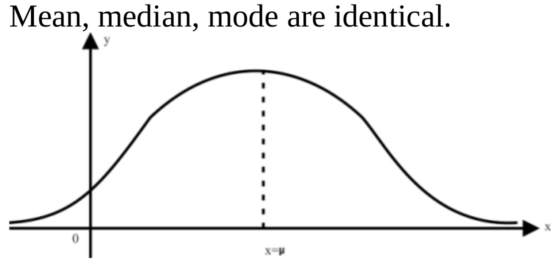
1. The graph of the normal distribution μ = f(x). xy plane is known as normal curve.
2. The curve is bell shaped symmetric about curve x = μ
3. Area under normal curve represents total population.
4. Mean, Median and Mode of the distribution coincide. So normal curve is unimodel.
5. x – axis is an asymptote to the curve.

IMPORTANCE OF NORMAL DISTRIBUTION:

1. Most of the distributions occurring in practice example, Binomial, Poisson, and Hyper geometric distribution etc. can be approximated by Poisson distribution.
2. Many of the distribution of sample static, the distribution of sample mean, sample variance etc tends to normality for large sample can be best studied with help of normal curve.
3. The entire theory of small sample tests based on the fundamental assumption that the parent population from which sample have been drawn follow normal distribution.
4. Theory of normal curves can be applied to the graduation of the curves which are not normal.
5. Normal distribution finds large application in statistical quality control industry of setting control limits.

CHIEF CHARACTERISTICS OF NORMAL DISTRIBUTION:

1. Mean, median, mode are identical.



2. The curve is smooth, regular, bell shaped symmetric about line x = μ.
3. Curve one maximum value at x = μ
4. Curve ca extends from -∞ to ∞
5. Total area under the curve above x – axis is unity.
6. Area between μ – σ andμ + σ is 68.27%.
7. μ - 2σ and μ + 2σ is 95.47.
8. μ - 3σ and μ + 3σ is 99.73.
9. Area bounded by the curve with X – axis and any two ordinates equal to probability for interval value.

AREA UNDER NORMAL CURVE, NORMAL PROBABILITY INTEGRAL:

Take $z = \frac{x-\mu}{\sigma}$

Total area under the curve is divided into two equal parts. Left hand side area and right hand side area. The area between $z = 0$ and any other can be obtained from table.

$$P(\mu < X < x_1) = \int_{\mu}^{x_1} f(x)dx = \frac{1}{\sigma\sqrt{2\pi}}\int_{\mu}^{x_1} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \frac{1}{\sqrt{2\pi}}\int_0^{z_1} e^{-\frac{z^2}{2}}dz$$

$$\left(z = \frac{x-\mu}{\sigma}\right)\left(z_1 = \frac{x_1-\mu}{\sigma}\right)$$

$$P(\mu < X < x_1) = P(0 < z < z_1) = \frac{1}{\sqrt{2\pi}}\int_0^{z} e^{-\frac{z^2}{2}}dz = A(z_1)$$

$Q(z) = \frac{1}{2\pi}e^{-\frac{z^2}{2}}$is probability function for normal curve.

$A(-z) = A(z)$

HOW TO FIND PROBABILITY DENSITY FOR NORMAL CURVE:

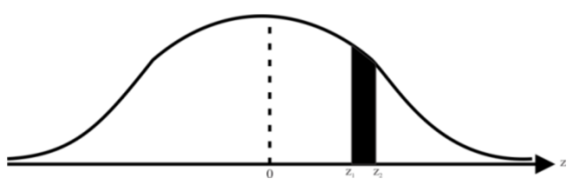
* Probability that normal variate with mean μ, S.D. σ, lies between two specific values $x_1\ and\ x_2,\ x_1 < x_2$, are obtained using area under normal curve $P(x_1 < X < x_2)$

Step 1: Perform the changes of scale $z = \frac{x-\mu}{\sigma}$ and find $z_1\ and\ z_2$ for distinct value of x.

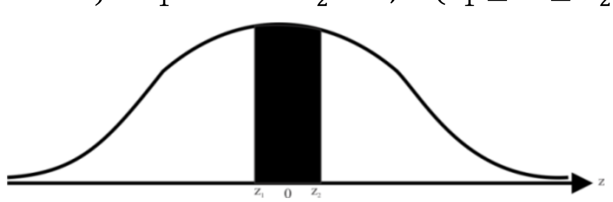
Step 2: (a) To find $P(x_1 \le X \le x_2) = P(z_1 \le Z \le z_2)$

Case i) If both $z_1\ and\ z_2$ are positive (or both negative) then

$$P(x_1 \le X \le x_2) = \left|\underbrace{A(z_2)}_{0\ to\ z_2} - \underbrace{A(z_1)}_{z_1 to\ z_2}\right|$$



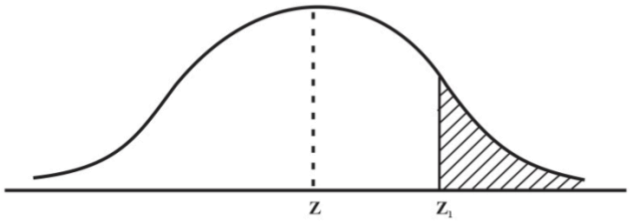
Case ii) If $z_1 < 0\ and\ z_2 > 0,\ P(x_1 \le X \le x_2) = A(z_2) + A(z_1)$



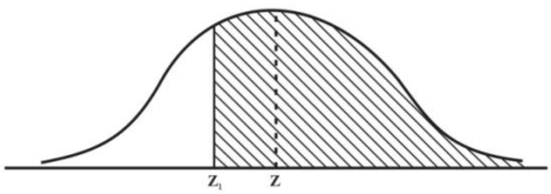
(b) To find $P(Z > z_1)$

Case i) $z_1 > 0$

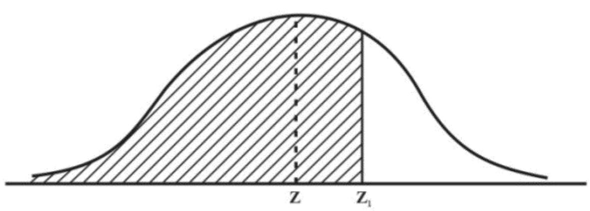
$P(Z > z_1) = 0.5 - A(z_1)[\because\ \ P(Z < 0) = P(Z > 0) = 1/2]$



Case ii) If $z_1 < 0$
$P(Z > z_1) = 0.5 + A(z_1)$



(c) To find $P(Z < z_1) = 1 - P(Z > z_1)$
Case i) If $z_1 > 0$ then
$P(Z < z_1) = 1 - P(Z > z_1) = 1 - (0.5 - A(z_1))$
$P(Z < z_1) = 0.5 + A(z_1)$



Case ii) If $z_1 \leq 0$ then
$P(Z < z_1) = 1 - P(Z > z_1) = 1 - (0.5 + A(z_1))$
$P(Z < z_1) = 0.5 - A(z_1)$

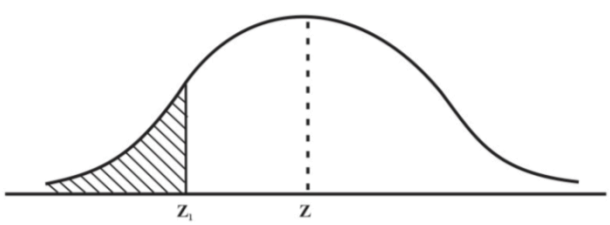
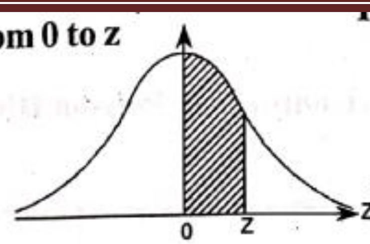


Table – 3

Areas under the Standard Normal Curve from 0 to z

$$Z=\frac{x-\mu}{\sigma}$$



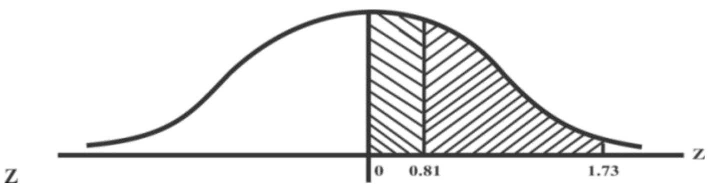
z	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1256	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1916	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2649
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4654	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4979	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

PROBLEMS ON NORMAL DISTRIBUTION:

1. For a normal distribution variate with mean 1 and S.D. 3, find the probability that i) $3.43 \leq x \leq 6.19$ ii) $-1.43 \leq x \leq 6.19$

Sol: $\mu = 1, \sigma = 3$

i) When $x = 3.43, Z = \frac{x-\mu}{\sigma} = \frac{3.43-1}{3} = \frac{2.43}{3} = 0.81 = z_1$



When $x = 6.19, Z = \frac{x-\mu}{\sigma} = \frac{6.19-1}{3} = \frac{5.19}{3} = 1.73 = z_2$

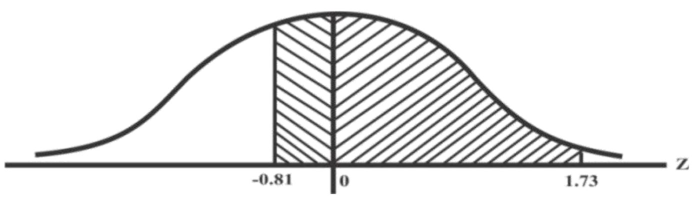
$P(3.43 \leq x \leq 6.19) = P(0.81 \leq Z \leq 1.73)$

$= A(z_2) - A(z_1) = A(1.73) - A(0.81)$ (From table)

$= 0.4582 - 0.2910 = 0.1672$

ii) $-1.43 \leq x \leq 6.19$

When $x = -1.43, Z = \frac{x-\mu}{\sigma} = \frac{-1.43-1}{3} = \frac{-2.43}{3} = -0.81 = z_1$



When $x = 6.19, Z = \frac{x-\mu}{\sigma} = \frac{6.19-1}{3} = \frac{5.19}{3} = 1.73 = z_2$

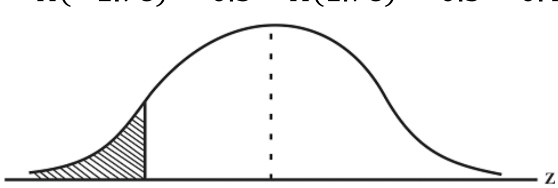
$P(-1.43 \leq x \leq 6.19) = P(-0.81 \leq Z \leq 1.73) = A(z_2) + A(z_1) = A(1.73) + A(-0.81)$

$= 0.4582 + 0.2910 = 0.7492$

2. If X is a normal variate find
   i) To the left of $Z = -1.78$
   ii) To the right of $Z = -1.45$
   iii) Corresponding to $-0.80 \leq Z \leq 1.53$
   iv) To the left of $Z = -2.52\ and\ to\ the\ right\ of\ Z = 1.83$

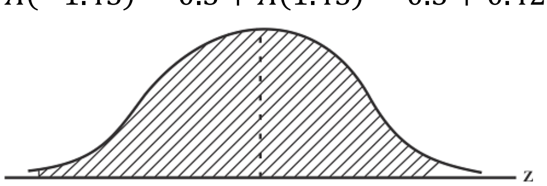
Sol: i) Required area =

$= A(-1.78) = 0.5 - A(1.78) = 0.5 - 0.4625\ (from\ table) = 0.0375.$



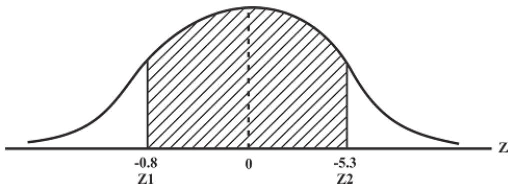
ii) To the right of $Z = -1.45$

$A(-1.45) = 0.5 + A(1.45) = 0.5 + 0.4256 = 0.9256$



iii) Area corresponding to $-0.80 \leq Z \leq 1.53$

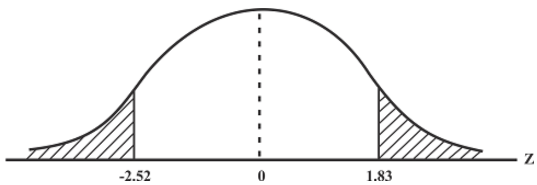
$A(0.8) + A(1.53) = 0.2881 + 0.4370 = 0.7251$



iv) Area to the right of $Z = -2.52$ *and to the right of* $Z = 1.83$

$[0.5 - A(2.52)] + [0.5 - A(1.83)] = (0.5 - 0.4941) + (0.5 - 0.4664)$

$= 0.0059 + 0.0336 = 0.0395.$



3. X is normally distributed with mean 8, S.D. = 4. Find i) $P(5 \leq X \leq 10)$, ii) $P(X \leq 15)$

Sol: Let X be a normal variate,

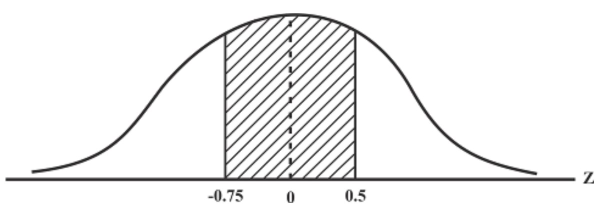
$Z = \frac{x-\mu}{\sigma}$

i) $P(5 \leq X \leq 10)$

When $X = 5, z_1 = \frac{5-8}{4} = -\frac{3}{4} = -0.75$

When $X = 10, z_2 = \frac{10-8}{4} = \frac{1}{2} = 0.5$

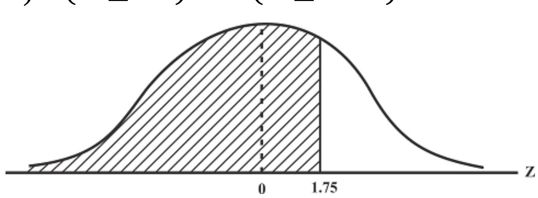
$P(5 \leq X \leq 10) = P(-0.75 \leq Z \leq 0.5)$



$= P(-0.75 \leq Z \leq 0) + P(0 \leq Z \leq 0.5) = A(0.75) + A(0.5)$

$= 0.2734 + 0.1915 = 0.4649$

ii) $P(X \leq 15) = P(Z \leq 1.75)$



For $X = 15,\ Z = \frac{X-\mu}{\sigma} = \frac{15-8}{4} = 1.75$

$= 0.5 + a(1.75) = 0.5 + 0.4599 = 0.9599$

4. Suppose the weight of 800 students are normally distributed with mean = 140 pounds, standard deviation 10 pounds, find number of students whose weights are

i) Between 138 and 148 pounds ii) more than 152 pounds.

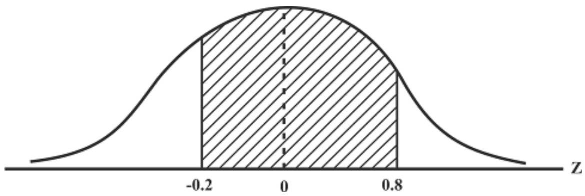
Sol: Mean $\mu = 140,\ \sigma = 10, n = 800$

$Z = \frac{x-\mu}{\sigma}$

i) Between 138 and 148 pounds $= P(138 \leq X \leq 148)$

$x_1 = 138, z_1 = \frac{138-140}{10} = -0.2,\ z_2 = \frac{148-140}{10} = 0.8$

$P(-0.2 \le Z \le 0.8) = A(0.2) + A(0.8)$

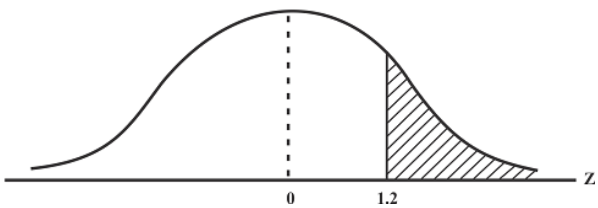


$= 0.0793 + 0.2881 = 0.3674$

No. of students = $0.3674\ \times 800 = 293.9 \cong 294$

ii) More than 152 pounds

$Z = 1.2 \quad P(X > 152) = P(Z > 1.2) = 0.5 - A(1.2)$



$= 0.5 - 0.3849 = 0.1151$

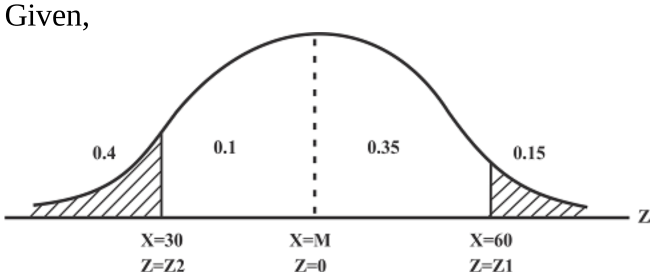
No. of students = $0.1151 \times 800 = 92.08 = 92$

5. The marks obtained in statistics in a certain examination found to be normally distributed. If 15% of the marks of the students is $(\geq)$ 60, 40% of the students < 30 marks, find mean, S.D?

Sol: $P(X \ge 60) = 0.15$

Since $0.15 < \frac{1}{2}$ is to the right of Z, Z must be positive such that area from 0 to Z is $0.5 - 0.15 = 0.35$ corresponding to Z value from table is 1.04 (in reverse we have to see i.e., A(1.04)=0.350)

Given,



$P(X < 30) = 0.4 < \frac{1}{2},\ Z$ Must be negative such that area from 0 to Z is 0.5 – 0.4 = 0.1

$= -0.25\ or\ 0.26$

Now, $\frac{60-\mu}{\sigma} = 1.04\ or\ \mu + 1.04\sigma = 60$

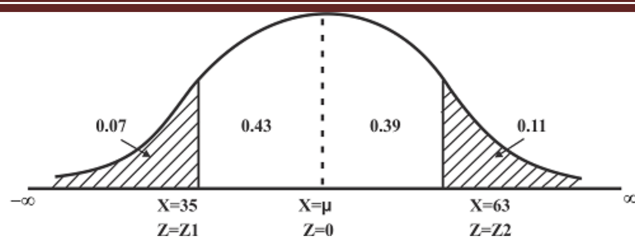
$\frac{30-\mu}{\sigma} = -0.25\ or\ \mu - 0.25\sigma = 30$

Solving the above two equations $\mu = 35.8,\ \sigma = 23.25$

6. Find mean and S.D. of normal distribution in which 7% of items are under 35 and 89% are under 63.

Sol: Let X be continuous random variable

Given$P(X < 35) = 0.07 < \frac{1}{2}$. So Z must be negative such that area from 0 to Z is 0.5 – 0.07 = 0.43 from table. $Z = -1.48$



Given that $P(X < 63) = 0.89 > \frac{1}{2}$, Z must be positive, area from 0 to Z is 0.89 – 0.5 = 0.39 = 1.23 from table.

So, $Z = \frac{x-\mu}{\sigma} = \frac{35-\mu}{\sigma} = -1.48 \;\Rightarrow\; \mu - 1.48\sigma = 35$

For 63, $\frac{63-\mu}{\sigma} = 1.23 \;\Rightarrow\; \mu + 1.23\sigma = 63$

Solving the above equations, we get

$\mu = 50.3,\ \sigma = \ 10.33$

7. A sales tax officer has reported that the average sales of 500 business that has deal with during a year is 36,000 with a S.D. of 10,000. Assume that sales of this person is normally distributed. Find i) the number of business as the sales of while are Rs 40,000.
ii) The percentage of business the sales of while are likely to range between Rs 30,000 and Rs 40,000.

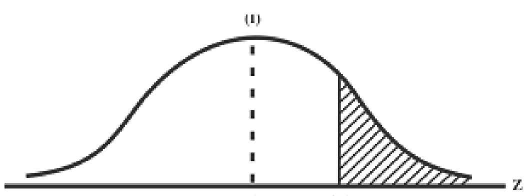
Sol: Mean $\mu = 36{,}000, N = 500, \sigma = 10{,}000$

We know that $Z = \frac{x-\mu}{\sigma} = \frac{x-36000}{10000}$

When $x = 40{,}000, z_1 = \frac{40000-36000}{10000} = \frac{4000}{10000} = 0.4$

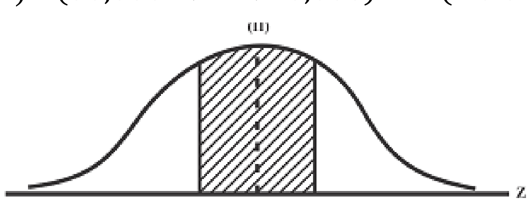
When $x = 30{,}000,\ z_2 = \frac{30000-36000}{10000} = -\frac{6000}{10000} = -0.6$

i) $P(X > 40000) = P(Z > 0.4)$



$= 0.5 - A(0.4) = 0.5 - 0.1554 = 0.3446 = 34.4\%$

ii) $P(30{,}000 < X < 40{,}000) = P(-0.6 < Z < 0.4)$



$= 0.2258 + 0.1554 = 0.3812 = 38.12$

8. What is the probability that X will be between 75 and 78 if a random sample of size 100 is taken from an infinite population has mean 76, variance 256.

Sol: Let X be normally distributed

$n = 100, \mu = 76,$

$\sigma^2 = 256 \Rightarrow \ \sigma = 16$

Standard normal variable $Z = \frac{X-\mu}{\sigma/\sqrt{n}} = \frac{X-76}{16/\sqrt{100}}$

When $X = 75,$

$z_1 = \frac{75-76}{16/\sqrt{100}} = -0.625$

When $X = 78$,

$z_2 = \frac{78-76}{16/\sqrt{100}} = 1.25$

$P(75 < \bar{X} < 78) = P(-0.625) < Z < (1.25)$

$= 0.3944 + 0.2324 = 0.6268$

9. A normal population has mean 0.1 and S.D. 2.1. Find probability that the mean of simple sample of 900 member will be negative.

Sol: $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$; $\mu = 0.1,\ \sigma = 2.1$

$z = \frac{\bar{x}-0.1}{2.1/\sqrt{900}} = \frac{\bar{x}-0.1}{0.07} = \frac{\bar{x}}{0.07} - 1.43$

$\bar{x}$is negative if $Z < -1.43$

$P(\bar{x} < 0) = P(Z < -1.43) = P(2 > 1.43)$

$= 0.5 - A(1.43) = 0.5 - 0.4236 = 0.0764$

$\Rightarrow\ \bar{x} = 0.0764$

RELATED PROPERTIES:

If n is large and p and q are small close to zero, the binomial distribution can be closely approximated to normal distribution. For this consider two cases

Case i) When $p = q = \frac{1}{2}$

B.D. can be approximated to N.D.

$Z = \frac{X-np}{\sqrt{npq}}\ where\ np = mean\ of\ B.D., \sqrt{npq} = variance\ of\ Binomial\ Distribution$

Consider No. of success x ranges from $x_1\ to\ x_2$ then probability of getting $x_1\ to\ x_2$ success is given by $\sum_{k=x_1}^{x_2}\ {}^{n}C_k p^k q^{n-k}$.

Suppose $z_1,\ z_2$ are values of Z corresponding to $x_1,\ x_2$

$P(x_1 < X < x_2) = P(z_1 < Z < z_2) = \int_{z_1}^{z_2} \emptyset(Z)dz$

We can find values using Normal Distribution.

Case ii) When $p \neq q$

For any success x real class and trial is $\left(x - \frac{1}{2}, x + \frac{1}{2}\right)$

$z_1\ corresponds\ to\ LL\ of\ x_1\ and\ z_2\ to\ LL\ of\ x_2$

$z_1 = \frac{\left(x_1-\frac{1}{2}\right)-\mu}{\sigma}, = \frac{x_1-\frac{1}{2}-np}{\sqrt{npq}}$

$z_2 = \frac{\left(x_1+\frac{1}{2}\right)-\mu}{\sigma} = \frac{x_1+\frac{1}{2}-np}{\sqrt{npq}}$

Required probability $= \int_{z_1}^{z_2} \emptyset(Z)dz$

From table we can find the values.

PROBLEMS ON RELATED PROPERTIES:

1. Find the probability that out of 100 students between 70 and 85 inclusive will pass an examination given that the chances of passing is 0.8.

Sol: Given that $p = 0.8, n = 100, q =\ 0.2$

S.D. $\sigma = \sqrt{npq} = \sqrt{100 \times 0.8 \times 0.2} = \sqrt{16} = 4$

$\mu = np = 100 \times 0.8 = 80$

Now to find $P(70 \leq X \leq 85)$

$x = 70,\ z_1 = \frac{70-\frac{1}{2}-80}{4} = -2.63$

$x = 85, z_2 = \frac{85+\frac{1}{2}-80}{4} = 1.38$

Required area$= P(-2.63 < Z < 1.28)$

$= A(1.38) + A(2.63)$

$= 0.4162 + 0.4957 = 0.9119$

PRACTICE PROBLEMS:

BINOMIAL DISTRIBUTION:

1. In a binomial distribution consisting of 5 independent trails, probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively find parameter P of the distribution.
2. 20% of items produced from a factory are defective. Find probability that in a sample of 5 chosen at random.
3. It has been claimed that in 60% of all solar heat installations the utility bill is reduced by at least one third. Accordingly what are probabilities that the utility bill will be reduced by at least one third in (i) four of five installations (ii) at least four of five installations?

POISSON DISTRIBUTION:

1. Derive mean and variance of Poisson distribution.
2. If x is a Poisson variate such that 3P(X= 4) = $\frac{1}{2}$ P(x = 2) + P(x = 0). Find (i) The mean of x (ii) P(x≤2).
3. If 2% of light bulbs are defective find (i) at least one is defective (ii) Exactly 7 are defective (iii) P (1 < x < 8) in a sample of 100.
4. Fit a Poisson distribution to the following data.

x	0	1	2	3	4	5	Total
f	142	156	69	27	5	1	400

NORMAL DISTRIBUTION:

1. Derive median and mode of Normal Distribution.
2. The marks obtained in statistics in a certain examination found to be normally distributed. If 15% of students ≥ 60 marks, 40%, < 30 marks. Find the mean and standard deviation.
3. If the masses of 300 students are normally distributed with mean 68 kgs and standard deviation 3 kgs how many students have masses
   (i) Greater than 72 kg (ii) Less than or equal to 64 kg
   (iii) Between 65 and 71 kg inclusive.
4. The heights of 1000 students are normally distributed with mean of 174.5cm and S.D of 6.9cm assuming that the heights are recorded to the nearest half-cm how many of these students would you expect to have heights
   (i) less than 160cm (ii) Between 171.5 and 182.0 cms (iii) Greater than or equal to 188 cm.

****